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Topological Cross Sections and Multiplicity Distributions for $\overline{p}-n$ and $p-n$ Interactions at High Energies

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Abstract— We have studied the charged particles multiplicity distributions arising from $\overline{p}-n$ and $p-n$ collisions over the range of laboratory momenta from 50 to 400 GeV/c. The parton two fireball model based on an impact parameter analysis is adopted. Figures and calculations are provided to demonstrate good agreement between theoretical calculations and experimental data at different momenta.

Index Terms— Hadron-Hadron Interaction / Parton Model/ Multiparticle Production.

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1 Introduction

Models are provided for the hadron structure [1-3]. These include the three fireball model [4], quark model [5], fragmentation model [6, 7], and many others. The theories and ideas concerning multiparticle production go back to the late of 1930's with a significant interlude at Fermi's statistical theory of particle production [8]. Multiparticle production can be also modeled and described efficiently by studying the multiplicity distribution [9]. Several methods exist which investigate the multiplicity distribution of particles at high energy [10-13]. Among these are the multiplicity scaling [10,11], the statistical boot strap model [12], the two sources model [14], the negative binomial distribution [15], fireballs [16], strings [17], quark gluon plasma [18,19] and many others.

Parton two fireball model have been used in studying hadron-hadron, hadron-nucleus and nucleus-nucleus interactions [20, 21]. All these studies showed good predictions of the measured parameters [22- 24]. Section 2 presents the proton neutron and antiproton neutron interactions at high energies. Section 3 presents the multiparticle production in proton neutron and antiproton neutron collisions. Section 4 provides the average charged particles multiplicity. Section 5 presents C_q - moments of the charged particles multiplicity distributions. Section 6 provides the so-called KNO-Scaling. Section 7 presents the dispersion of the charged particles multiplicity distributions. Section 8 presents the results and conclusion.

2 $p^{\pm}-n$ Interactions At High Energies

According to the parton two fireball model [20, 21, 25], $\overline{p}-n$ and $p-n$ interaction will be characterized by the impact parameter and the corresponding overlapping volume. Let us assume that the two interacting hadrons at rest are spheres each of radius (R). Therefore the two colliding particles can interact strongly when the impact parameter is in the region from 0 → 2R.

Therefore, the statistical probability of any impact parameter (b) within an interval (db) is given by,

$$P(b)db = \frac{bdb}{2R^2} \tag{1}$$

Let us use a dimensionless impact parameter, X, defined as,

$$X = \frac{b}{2\,R}$$

Then, Eq. (1) can be rewritten as,

$$P(X)\ dX = 2X\ dX \tag{2}$$

where, $0 \le X \le 1$

Now we employ the overlapping volume, $V(b)$ as a clean cut [26] as,

$$V(b)db = (1-\frac{3b}{8R}-\frac{3b^2}{8R^2}+\frac{5b^3}{32R^3}) \tag{3}$$

In terms of the dimensionless impact parameter (X), the overlapping volume $V(X)$ can be given by,

$$V(X)=V_o\,(1-0.75X-1.5X^2+1.25X^3) \tag{4}$$

Then, the fraction of partons, $Z(X)$ participating in the interaction may be written as,

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$$Z(X)= \frac{V(X)}{V_0} = (1-0.75X-1.5X^2+1.25X^3) \qquad (5)$$

According to Eq. (2) and Eq. (5), the Z -function distribution can be given by,

$$P(Z)\,dz = 2XdX\,(-2.4375X-0.75X^{-1}+7.125X+0.75X^2 -9.375X^3+4.687X^4)^{-1} \qquad (6)$$

where, $0 \leq Z \leq 1$

From Eqs. (2, 5) and using least square fitting technique (LSFT), Z -function distribution can be written in the following form,

$$P(Z)dz = \sum_{k=-1}^{3} C_k Z^k dz \qquad (7)$$

where, C_k (k = -1, 0, 1, 2, 3) are free parameters to be calculated to produce a fitting between Eq. (6) and the curve drawn from Eq. (7). From such fitting procedure the obtained values for C_k are,

$$C_{-1} = 0.089,\ C_0 = 1.21,\ C_1 = -2.65,\ C_2 = 3.228$$

and $C_3 = -1.823$

3 Multiparticle Production In Proton-Neutron And Antiproton- Neutron Collisions

After the collision takes place, the partons within the overlapping volume stop in the center of mass system (CMS); their kinetic energy (K.E) will be changed into excitation energy to produce two intermediate states (fireballs). The produced fireballs will radiate the excitation energy into a number of newly created particles, which are mostly pions. We assume that each fireball will decay in its own rest frame into a number of pions with an isotropic angular distribution. The number of created pions will be defined by the fireball rest mass (M_f) and the mean energy consumed in the creation of each pion (ε).

The energy available for the creation of pions from each fireball will be,

$$M_f - m = T_o Z(X) \qquad (8)$$

where, T_o is the kinetic energy of the incident proton in CMS and given by, $T_o = \frac{Q}{2}$, Q is the total available kinetic energy in CMS.

The number of created pions (n_o) from each fireball will be given by,

$$n_o(Z) = \frac{Z(X)T_0}{\varepsilon} = \frac{Z(X)Q}{2\varepsilon} \qquad (9)$$

It is clear that Eq. (9) gives the total number of created particles (charged and neutral) as a function of the dimensionless impact parameter.

To get the charged particles multiplicity distribution, we have to assume some distribution for the charged particles (n_{ch}) in the final state of the interaction at any impact parameter out from the total created particles (n_o). We considered the new created particles from each fireball can be divided into a number of pairs. Each pair will be either charged or neutral to satisfy the charge conservation.

From equations (7) and (9), the total number of created particles distribution, $P(n_o)$ can be calculated from the following equation,

$$P(n_o) = \sum_{k=0}^{3} \left(\frac{1}{Q}\right)^{k+1} C_k \left[\frac{\left[2a(n_o+1)^2 + 2b(n_o+1)^2\right]^{k+1} - (2an_o^2+2bn_o)^{k+1}}{k+1}\right] + C_{-1} \ln\left[\frac{2a(n_o+1)^2 + 2b(n_o+1)}{2an_o^2 + 2bn_o}\right] \qquad (10)$$

We assume a binomial and Poisson distributions for the probability distribution for the creation of charged pion pairs from one fireball of the forms,

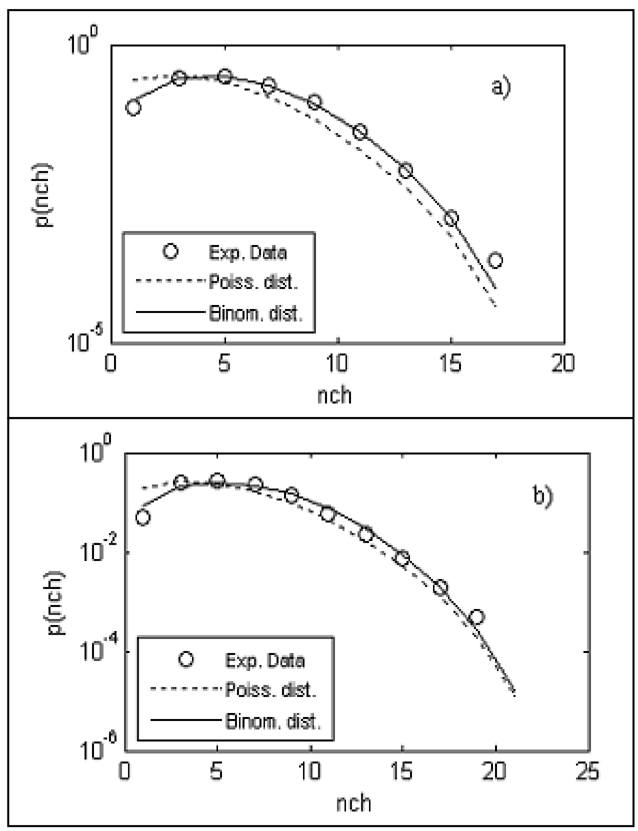
1) Binomial distribution of the form,

$$\psi(n_2) = \frac{N!}{n_2!(N-n_2)!} p^{n_2} q^{(N-n_2)} \qquad (11)$$

2) Poisson distribution of the form,

$$\psi(n_2) = \frac{N^{n_2}}{n_2!} p^{n_2} e^{-Np} \qquad (12)$$

where, N is the number of pairs of created particles from one fireball ($N = n_0/2$), n_2 the number of pairs of charged pions, p the probability that the pair of pions is charged, q the probability that the pair of pions is neutral.



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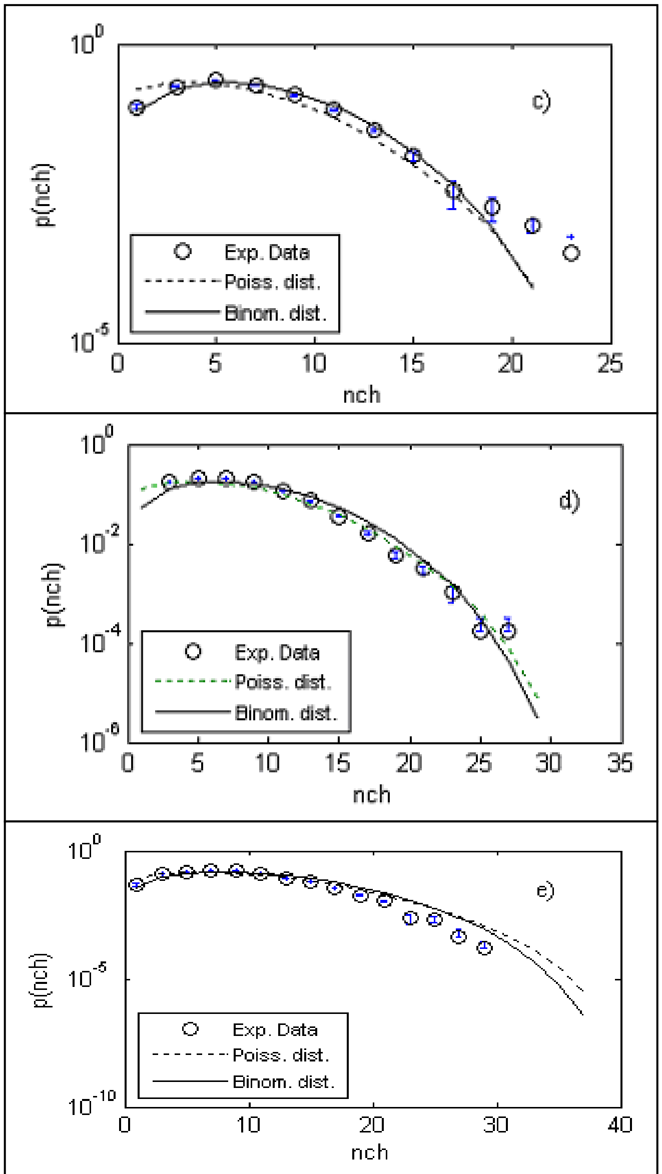

Fig. 1. Normalized multiplicity distribution of charged particles n_{ch} for $\bar{p}-n$ and $p-n$ collisions calculated according to the parton two fireball model as parameterized by Poisson and binomial distribution using Eq.(14) in comparison with the corresponding experimental data at a) 50, b) 80, c) 100, d) 200 and e) 400 GeV/c.

Therefore, the charged particles distribution from one fireball will be given by,

$$\phi(n) = \sum_{n_0} \psi(n_2) P(n_0) \qquad (13)$$

Then, the charged particles multiplicity distribution from the two fireballs will be,

$$P(n_{ch}) = \sum_{n=1}^{n_{ch}} \phi(n)\phi(n_{ch}-n) \qquad (14)$$

$$; n_{ch} = 2,4,6,........., Q/\varepsilon$$

We assume that ε increases with the multiplicity size, (N_0) as, $\varepsilon = a\, N_o + b$ where, a and b are free parameters which can be taken to be, a = 0.02, b= 0.27.

Charged particles multiplicity distributions have been calculated at $P_L = 50,80$ GeV/c for $\bar{p}-n$, $P_L = 100,200,400$ GeV/c for $p-n$ which are represented in fig. (1) a), b), c), d) and e) along with the corresponding experimental data [27-31].

We have also modified our calculations by changing Z and dZ as follows,

$$Z = \frac{1}{Q}(2an_0^2 + 2bn_0)\ , \ dZ = \frac{1}{Q}(4an_0 + 2b)dn_0$$

By substituting in Eq.(7), we will get the following equation,

$$P(n_o) = \sum_{k=0}^{3}\left(\frac{1}{Q}\right)^{k+1} c_k \left[\frac{\left[2a(n_o+1)^2 + 2b(n_o+1)\right]^{k+1} - (2an_o^2 + 2bn_o)^{k+1}}{k+1}\right] + c_{-1}\ln\left[\frac{2a(n_o+1)^2 + 2b(n_o+1)}{2an_o^2 + 2bn_o}\right] \qquad (15)$$

Therefore, the charged particles distribution from one fireball will be given by,

$$\varphi_1(n) = \sum_{n_0} \psi(n_2) P(n_o) \qquad (16)$$

Then, the charged particles multiplicity distribution from the two fireballs will be,

$$P_1(n_{ch}) = \sum_{n=1}^{nch} \varphi_1(n)\varphi_1(n_{ch}-n) \qquad (17)$$

$$; n_{ch} = 2,4,6,....., Q/\varepsilon$$

We again assume that ε increases with the multiplicity size, (n_0) as, $\varepsilon = a\, n_o + b$ where, a = 0.01, b = 0.35 for $\bar{p}-n$ and a = 0.01, b = 0.44 for $p-n$.

Our new calculations have been calculated using different free parameters, a = 0.01, b = 0.35 for $\bar{p}-n$ and a = 0.01, b = 0.44 for $p-n$ interactions and are represented in fig. (2) a), b), c) ,d) and e) along with the corresponding experimental data [27-31].

4 The Average Charged Particles Multiplicity

From the charged particles multiplicity distributions, it is possible to calculate the average charged particles multiplicity by using the relation,

$$< n_{ch} >= \sum_{n_{ch}=2}^{Q|\varepsilon} n_{ch} P(n_{ch}) \qquad (18)$$

Thus, the multiplicity distributions of charged particles described above (Eqs.14, 17) are used to calculate the average charged particles multiplicity at different incident momenta. These calculations are represented in fig. (3) along with the

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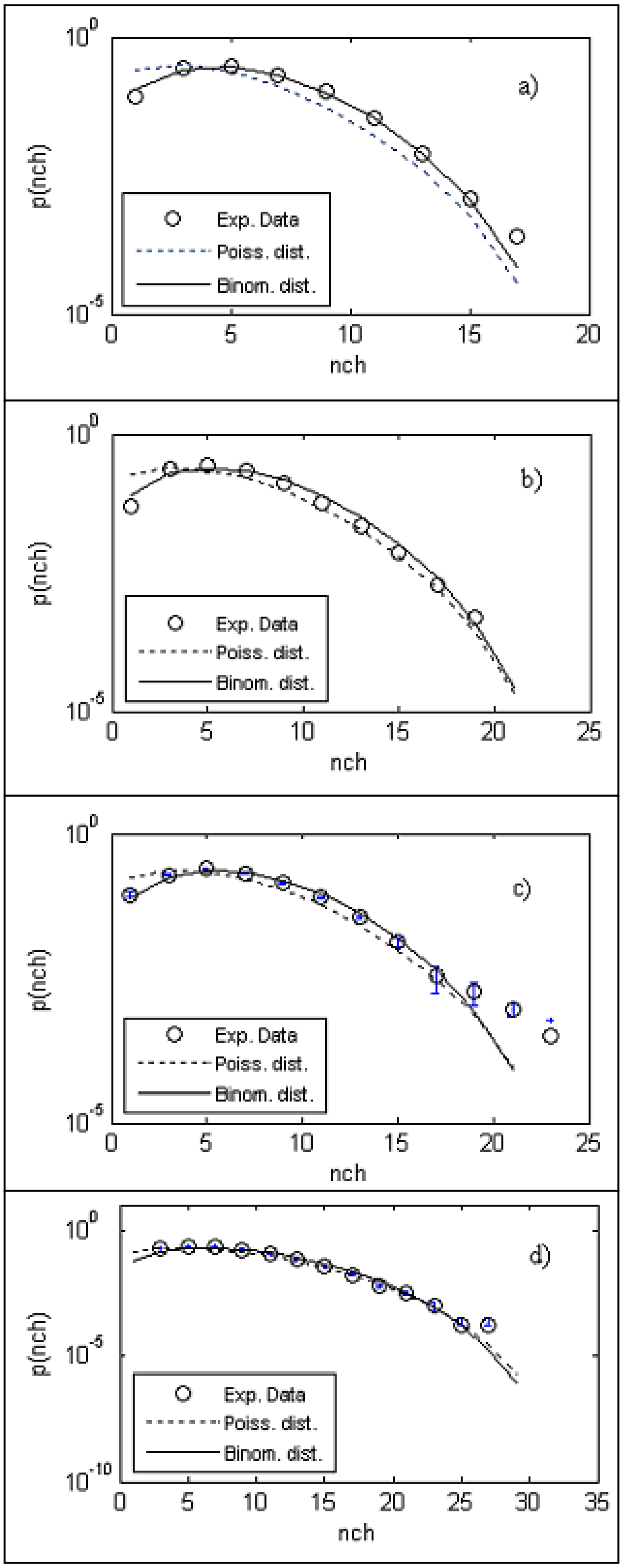
available experimental data [27-31] which shows good agreement with the experimental values.



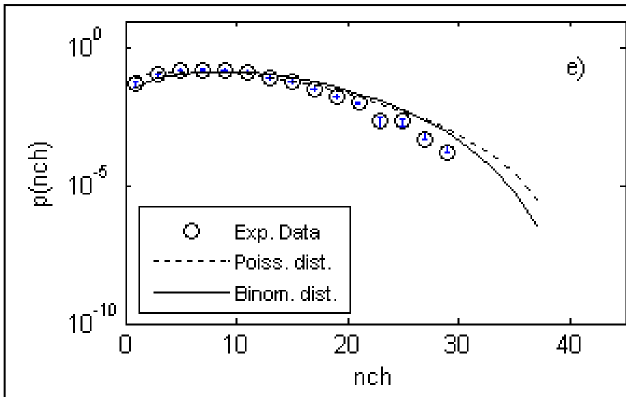


Fig. 2. Normalized multiplicity distribution of charged particles n_{ch} for $\bar{p}-n$ and $p-n$ collisions calculated according to the parton two fireball model as parameterized by Poisson and binomial distribution using Eq.(17) in comparison with the corresponding experimentaldata at a) 50, b) 80, c) 100, d) 200 and e) 400 GeV/c.

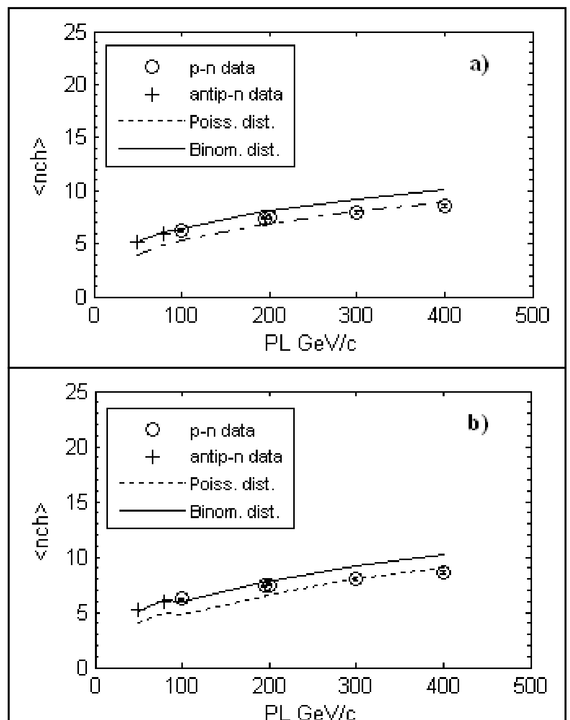


Fig. 3. Variation of average number of charged particles $<n_{ch}>$ for $\bar{p}-n$ and $p-n$ versus the incident momentum predicted by the parton two fireball model using a) Eq.14, b) Eq.17.

5 c_q -Moments of the Charged Multiplicity Distribution

The normalized moments c_q are defined by the relation,

$$c_q = < n_{ch}{}^q > / < n_{ch} >^q \tag{19}$$

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where q = 2, 3,……..,10.

Using the above relation, C_q -moments are calculated and represented in fig. (4) along with the available experimental data [32-38] which shows good fitting with the experimental data.

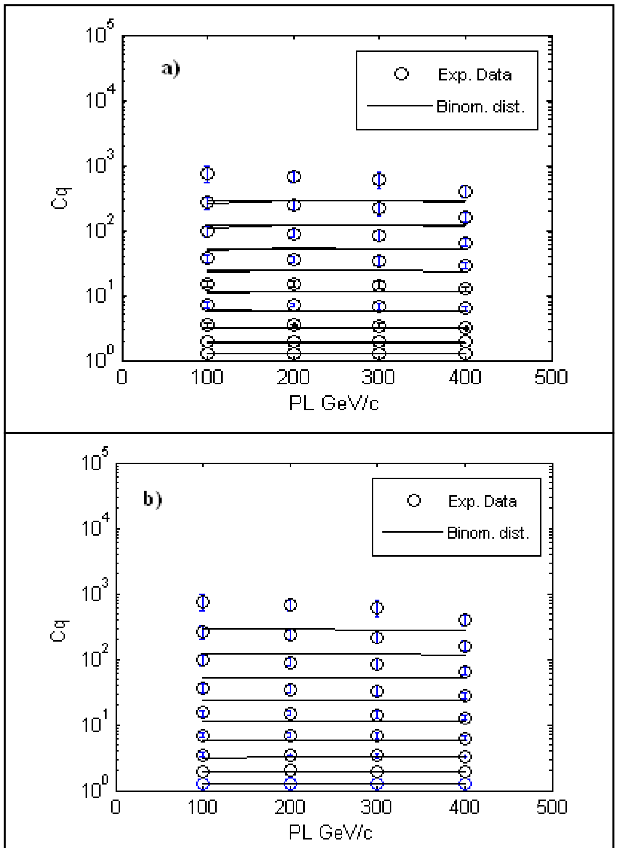

Fig. 4. The dependence of C_q moments for $p-n$ collisions at 100-400 GeV/c as a function of the incident momenta using a) Eq.14, b) Eq.17.

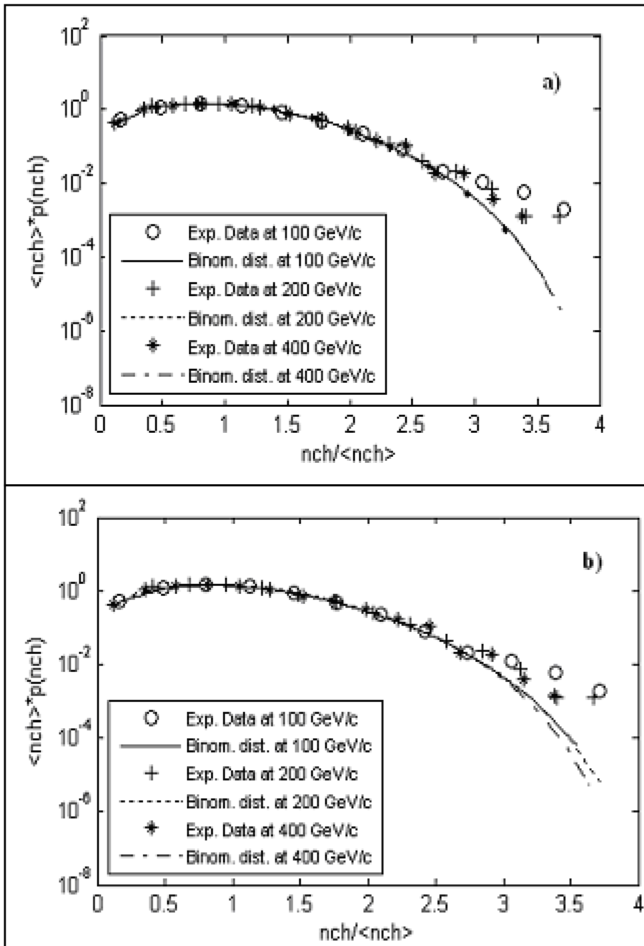

Fig. 5. Multiplicity distribution of charged particles n_{ch} for $p-n$ collisions at energies ranges 100-400 GeV/c using the parton two fireball model in the form of KNO-Scaling in comparison with the corresponding experimental data using a) Eq.14, b) Eq.17.

6 KNO-Scaling

An interesting feature of the topological cross section is the idea of KNO- scaling [12] suggested by Koba, Nielson and Olesson. . According to KNO - Scaling, if we plot the relation between multiplicity distributions of charged particles based on the above scheme multiplied by the average charged particles multiplicity $< n_{ch} > P(n_{ch})$ and the number of charged particles divided by the same quantity $n_{ch} / < n_{ch} >$ at different momenta, then, the relation must be energy independent. These calculations are represented in fig. 5. The obtained results show good agreement with the corresponding experimental data [27-31].

7 Dispersion of The Charged Multiplicity Distributions

The dispersion of the multiplicity distribution is defined as,

$$D = \left(< n_{ch}^2 > - < n_{ch} >^2 \right)^{1/2} \quad (20)$$

and the dispersion parameter is $< n_{ch} > / D$ which is found experimentally to be approximately constant with the increase in the incident momenta.

We have calculated the charged multiplicity moments, $< n_{ch}^2 >$, $< n_{ch} >^2$ and hence the dispersion parameter $< n_{ch} > / D$, the results obtained at different momenta are given in fig. (6) together with the available data [27-31]. It can be seen from the figure that the predictions are in an excellent agreement with observations.

8 Results And Conclusion

The charged particles multiplicity distributions, Eq. (14), are calculated for $\bar{p}-n$ and $p-n$ assuming ε is given by, $\varepsilon = a N_o + b$ where, a = 0.02, b= 0.27. The results of these calculations are represented in fig. (1) a), b), c), d) and e) along with the experimental data [27-31]which show good agreement with the corresponding experimental data. It can be seen from fig. (1) that the emission of secondary particles is assumed to follow a binomial distribution.

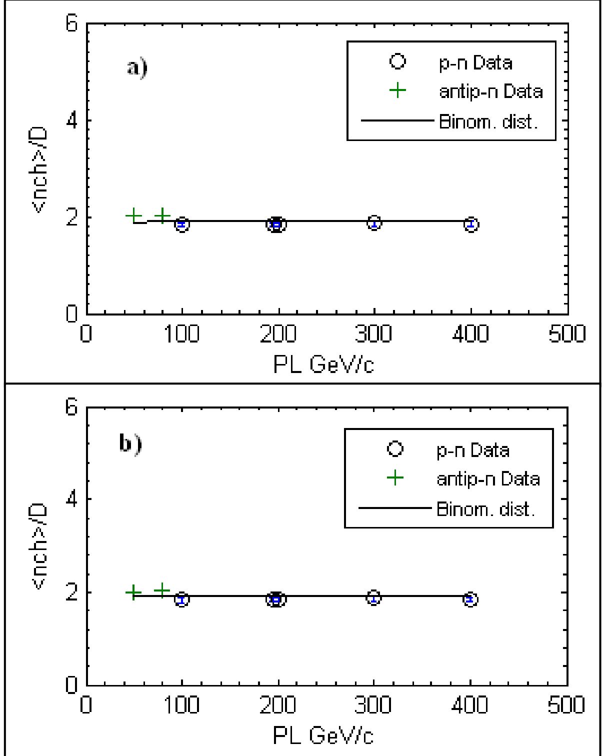

Fig. 6. Dispersion for $\overline{p}-n$ and $p-n$ versus P_L GeV/c using a) Eq.14, b) Eq.17.

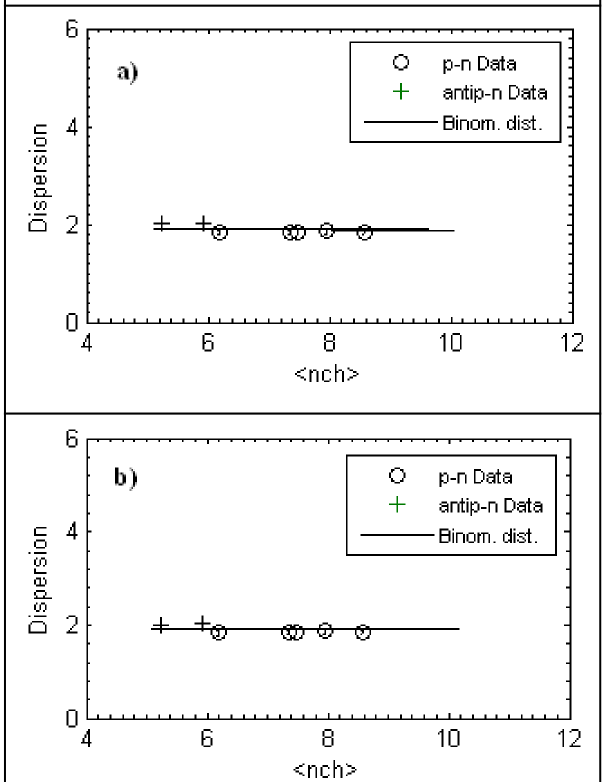

Fig. 7. Dispersion versus $<n_{ch}>$ using a) Eq.14, b) Eq.17.

We have also modified our calculations by changing Z formula and these calculations are representedin fig. (2) a), b), c), d) and e) along with the same experimental data [27-31]. We have also found some slight variations in comparison with fig.(1). Fig. (3) shows the variation of the average charged particles multiplicity $<n_{ch}>$ with various laboratory momenta (50, 80, 100, 200 and 400 GeV/c), from fig. (3) it can be seen that the dependence of $<n_{ch}>$ on laboratory momenta is in accordance with the experimental data [27-30]. Fig. (4) shows the dependence of C_q moments for $p-n$ collisions at 100-400 GeV/c on incident momenta and shows also a good fitting between our calculations and the corresponding experimental data [32-38]. Fig.(5) views the multiplicity distribution of charged particles n_{ch} for $p-n$ collisions at momenta ranges 100 - 400 GeV/c using the parton two fireball model in the form of KNO-Scaling in comparison with the corresponding experimental data[27-31] and that scaling is clearly energy independent. Figs. (6, 7) show the dependence of dispersion on incident momenta and the correlation between dispersion and the average multiplicity and the experimental data [27-31] are consistent with our predictions.

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